

The Mathematics of Diversification

Calculating the Correlation of Investments Within a Portfolio



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In a previous blog post, we discussed the key role that correlations play in the success or failure of a diversification strategy. In this post, we dig a little bit deeper to see how the numbers play out. Although it is certainly possible to go overboard with the mathematics when it comes to portfolio optimization, we will keep it simple and straightforward in this discussion.

When Harry Markowitz started an intellectual revolution with his seminal paper "[Portfolio Selection](#)" in 1952, one of his key contributions was quantifying the impact that correlations between pairs of investments had on reducing a portfolio's overall risk. The idea was that the contribution to overall portfolio risk that a single investment brought to a portfolio was not just its weight in the portfolio, but how it interacted with the other constituents of the portfolio.

This interaction is measured by correlation.

Technically speaking, correlation is a scaled version of covariance. Covariance measures how two variables change in relationship to one another. Correlation normalizes covariance to a scale ranging from -1.0 to +1.0.

Calculating how much return the various investments contribute to a portfolio return is quite straightforward. It is simply the weighted average of the returns of each individual investment. The generalized formula looks like this:

$$R_p = \sum_i w_i R_i$$

But for simplicity's sake let's assume a two-asset portfolio, in which case the formula is this:

$$R_p = w_A R_A + w_B R_B$$

Pretty simple, right? The weight of one asset (w_A), multiplied by its return (R_A) plus the weight of the other asset (w_B), multiplied by its return (R_B).

The formula for calculating an investment's contribution of risk to a portfolio is more complicated, but that's actually a good thing. Correlation is the "X-factor" that has the potential to lower a portfolio's overall volatility. As before, "w" is the weight and the new terms "σ" and "ρ" signify the standard deviation and covariance, respectively. The formula for a portfolio's variance is:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

Again, the generalized formula looks intimidating, but breaking it out into a simple two-asset portfolio makes it easier to understand.

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}$$

The first two terms in the equation represent the stand-alone contribution of the risk of assets A and B to the portfolio. Because the weights will be represented by a decimal and because they are being squared, the impact of the stand-alone risk will always be less than the weight. For example, an 80% weight squares out to .64, and a 40% weight squared would be .16.

This makes the third term, the combined risk of assets A and B, pretty important. The symbol for correlation is highlighted in red. By working through the formula, it becomes evident just how important that piece of the puzzle is.

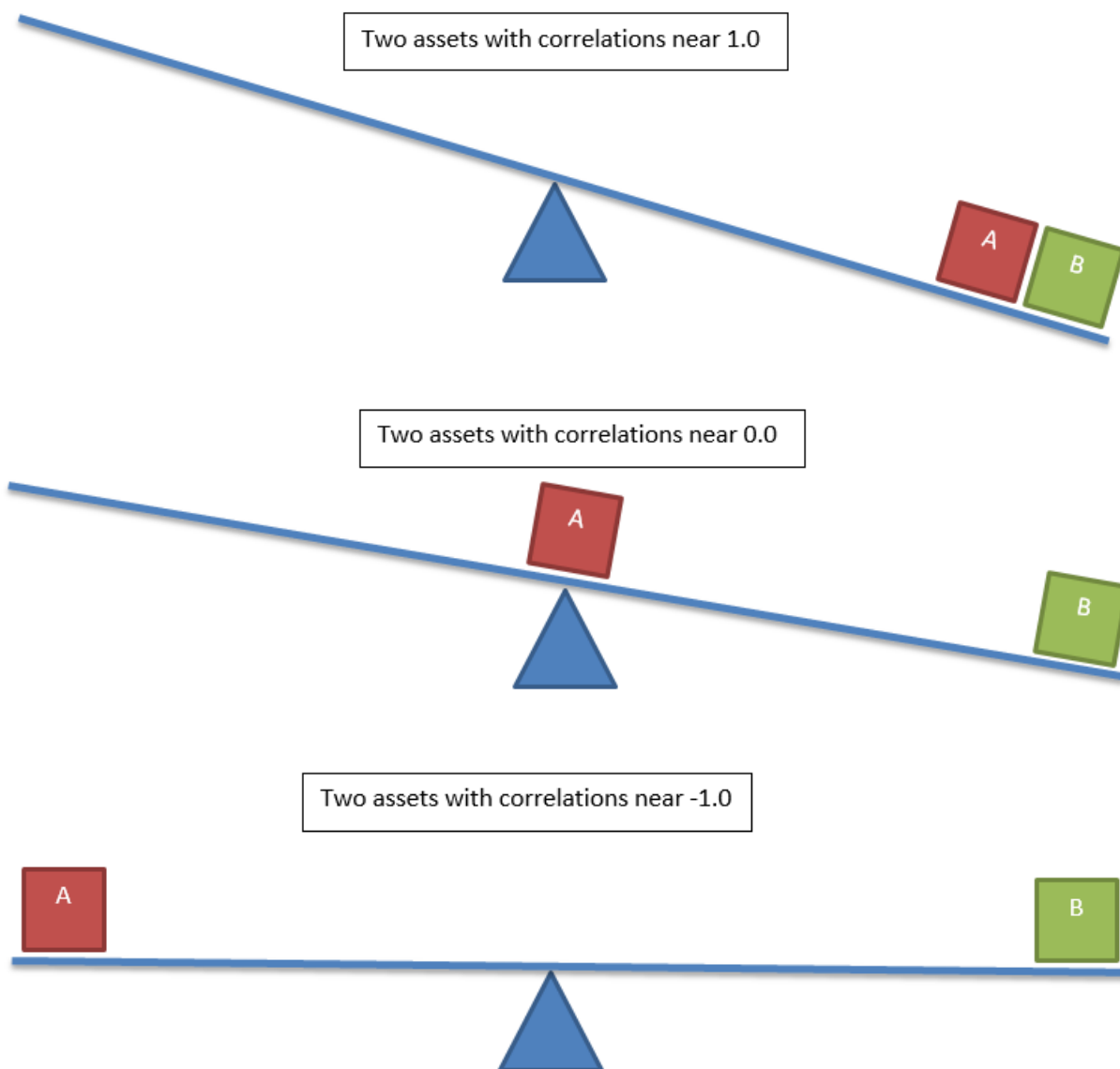
- If the correlation between A and B is 1.0, then overall variance is not reduced at all.
- If the correlation between A and B is 0.0, the whole third term of the equation will be erased and the variance of the portfolio will simply be the first two terms – a big improvement.
- If correlations are negative, then the whole third term becomes negative, and we actually get to subtract the third term from the first two.

That's when risk reduction really comes in to play.

By walking through the formula above, we see how important correlation is to the overall risk equation.

Unfortunately, this is where most diversification strategies fall apart.

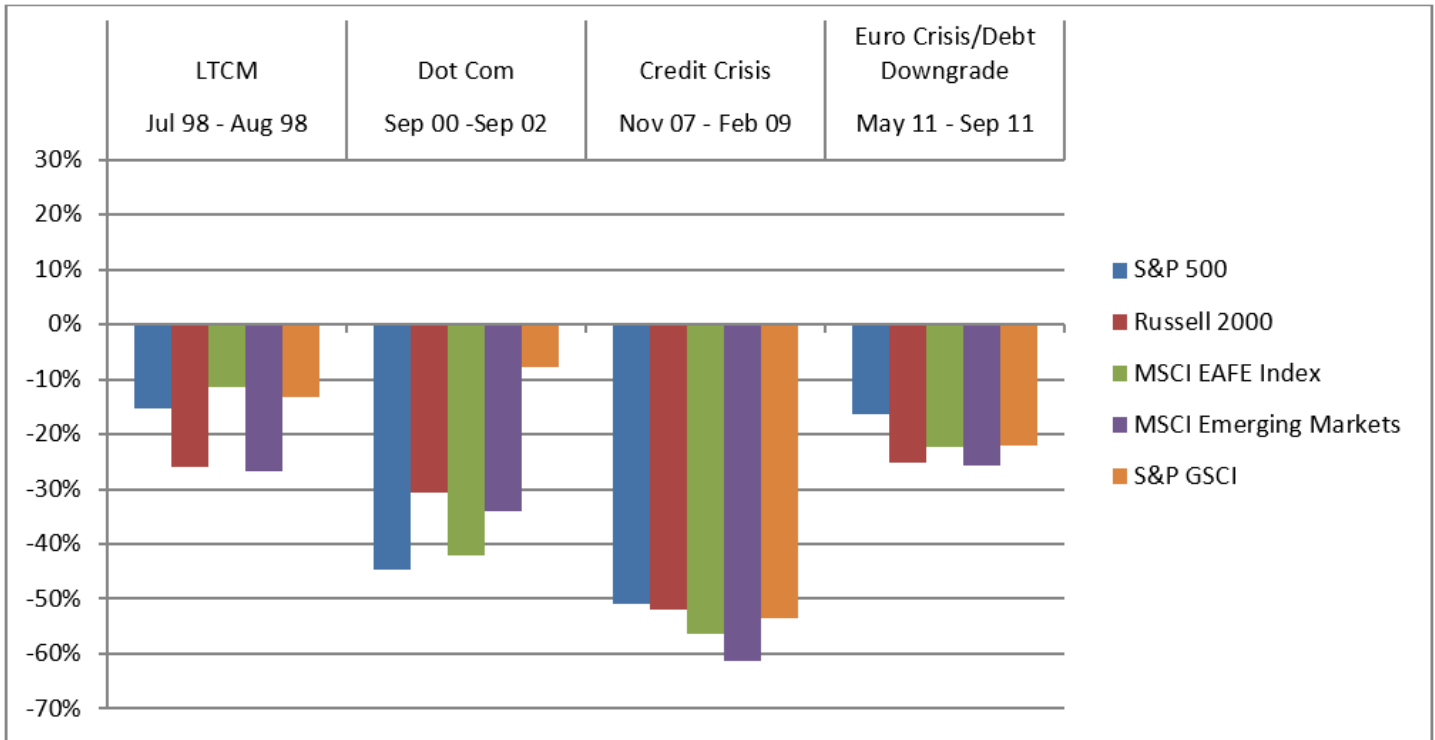
Although this post is based on mathematics, the old saying “a picture is worth a thousand words” holds true. In the three illustrations below, we attempt to show the how high correlations fail to balance risk and how low correlations help to even the scales.



As discussed in our previous blog post [“Serving Up Diversification – The Importance of Correlations to Diversification”](#), many investors pursued what we call “false diversification” strategies, where the correlations might have been 0.80, 0.90, or higher. Plugging such numbers in to our formula above does little to address overall portfolio risk.

Even if portfolios were built using asset classes or styles where the long-term correlations were low, the unfortunate reality is that correlations spiked in the midst of a crisis. During a major market sell-off the carnage tends to be widespread as panicked investors hit the “sell” button indiscriminately. The benefits of diversification tend to evaporate right when they are most needed. In the example below we see how some asset classes used for diversification

purposes actually performed worse than the core S&P 500 during major market downturns.



Source: Zephyr StyleADVISOR

Investors therefore need a way to deal with risk directly.

The Swan Defined Risk Strategy was designed to have different components contributing to performance at different points in time. Primarily the DRS is always invested in an underlying asset, comprised of all 9 Select Sector SPDRs. Then the DRS applies a hedge position on the full notional value of that underlying asset, in the form of long-term put LEAPS, which has a negative correlation to the equity portion. The premium-selling, income trades are designed to be market neutral, with the intent of having a zero correlation to market direction. By combining our equity, hedge, and income components, we believe the DRS is a better way of implementing a truly diversified strategy.

To learn more about Swan’s DRS investment approach and how this approach has fared in the past, please contact Swan at **970-382-8901**.

About the Author:



Marc Odo, CFA®, CAIA®, CIPM®, CFP®, Director of Investment Solutions, is responsible for helping clients and prospects gain a detailed understanding of Swan's Defined Risk Strategy, including how it fits into an overall investment strategy. Formerly Marc was the Director of Research for 11 years at Zephyr Associates.

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