SWAN GLOBAL INVESTMENTS

MATH MATTERS

Rethinking Investment Returns & How Math Impacts Results

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swanglobalinvestments.com

RETHINKING INVESTMENT RETURNS

Every investor dreams of finding the holy grail of investing: a no-risk, high return investment. But as everyone should know, such an investment does not exist. Obviously, all investments have some risk in some shape or form. So if one cannot invest in a riskless high return investment, what is the next best thing? What types of return lead to solid investment growth? Is it frequent large gains with only an occasional big loss? Is it smaller, flat, steady returns such as those sometimes found in fixed income? Is it frequent modest gains with relatively few, small losses? These questions unpack how the math behind these various types of return streams affect the final outcome.

Investors have many misconceptions around the math behind equity investment returns and hedged equity returns. Many investors make their investment decisions based on emotions like fear and greed and fail to fully grasp the math that drives successful long-term results. It's important to help them understand the core mathematical principles driving investment returns so they can make better investment decisions. These core principles are often overlooked, ignored, or misunderstood and will be explored in this paper for the purpose of strengthening the decision-making process.

The paper will focus on four core, interconnected mathematical principles, all of which are instrumental to achieving better investment results over time. They are:

1. The importance and power of compounding
2. The value of avoiding large losses
3. The importance of variance drain
4. The importance of a non-normal distribution of returns

The purpose is two-fold. First, Swan believes it is important that investors have a better understanding of how different equity investments ebb and flow and grow over time in order to properly assess them. Second, many investors familiar with passive or active equity investments sometimes have certain misperceptions regarding hedged equity strategies. This is mainly because a hedged equity approach generally underperforms during bull markets and tends to outperform in bear markets.

The final outcome is greatly impacted by the volatility of the experience along the way. Whether or not an investor can stay the course with an investment or survive withdrawing money from their account depends on the volatility of the portfolio. High volatility portfolios tend to lead to poor, emotionally driven decisions because investors have a difficult time staying invested.

With hedged equity approaches, investors will generally sacrifice some upside potential during bull markets in exchange for some portfolio protection on the downside. Many investors hear the phrase “give up some of the upside” and think that mathematically the strategy will always underperform the equity markets. On the contrary, if investors have a proper understanding of the math behind investment returns and what impacts them the most, they will see why and how hedged equity returns can be a superior investment approach compared to traditional management despite sacrificing some upside.

FOUR MOST IMPORTANT MATHEMATICAL PRINCIPLES TO GROWING WEALTH

1. The Power of Compounding

Albert Einstein supposedly once said that the most powerful force in the universe is compound interest. The principle of compound growth can be defined as the power of exponential growth, that is, growth on growth. The concept of compound growth and its impact can be a difficult one to grasp, though. Why is compound growth so important and how does it impact the returns achievable with an investment?

The power of compounding is basically the snowball effect that happens when growth generates even more growth and continues to do so. You receive growth not only on your original investments, but also on any interest, dividends, and capital gains that have accumulated — thus, your money can grow faster and faster as time goes on. The late Dr. Albert Bartlett, a professor, author, and expert on arithmetic and exponential growth, painted an interesting picture as it relates to the power of compounding and exponential growth in one of his papers. An adaptation by economic analyst Chris Martensen explains Dr. Bartlett's analogy like this:

> Let's say we have a magic eye dropper that we use to place a single drop on the pitcher's mound at Fenway park. This drop doubles in size every minute. Nothing much seems to happen at first; the little pool of water is two drops, then four drops, and so on. After 45 minutes, only 3% of the stadium is filled with water. Yet, in five minutes, the remaining 97% of the stadium is full of water. This is one of the key features of compound growth: "With exponential growth in a fixed container, events progress much more rapidly toward the end than they do at the beginning" (Source: Martensen, Chris; "The Crash Course: The Unsustainable Future Of Our Economy, Energy, And Environment").

Although this visualization doesn't use money, it does show the incredible power of exponential growth and how growth on growth can have a slow and sneaky impact over time. In investing, the well-known "rule of 72" refers to a shortcut in estimating how long it would take to double your money based on taking 72 and dividing it by the compound annual growth rate. For example, with a 10% compounded annual return, your money would double in 7.2 years.

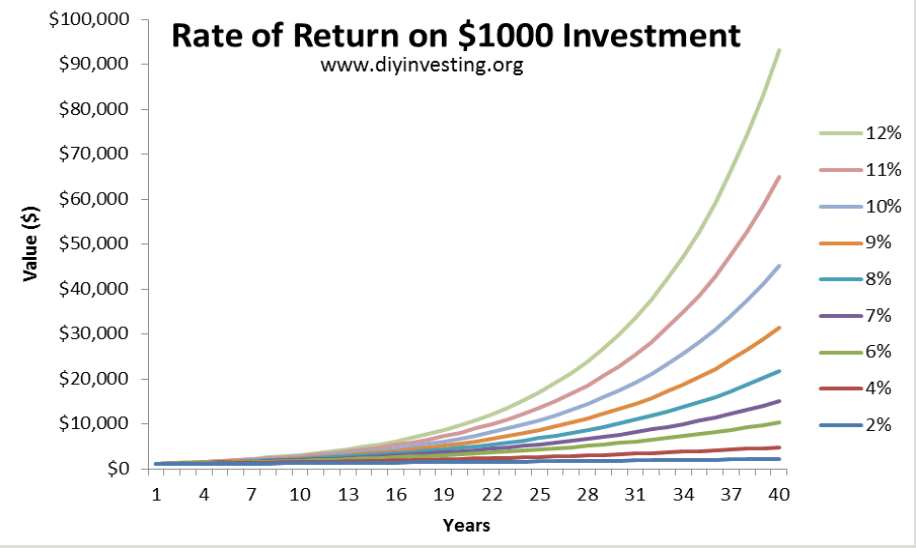
How long it takes to double your money with an investment strategy matters because the shorter the time period, the sooner the power of compounding kicks into high gear (see Exhibit 2 below). The sooner and steadier that growth occurs should lead to better long-term results. Conversely, lower rates of return and higher volatility will lead to lower long-term results.

Doubling Times for Different Rates of Steady Growth

Percent Growth Per Year	Doubling Time In Years
Zero	Infinity
0.5	144.0
1.0	72.0
2.0	36.0
3.0	24.0
4.0	18.0
5.0	14.4
10.0	7.2
20.0	3.6

Source: Swan Global Investments; calculations using rule of 72

The hypothetical graph below shows the power of compounding for an investment with no volatility. You can see how compound growth takes time to start to have an impact. Now imagine a large drawdown due to a bear market instead of the smooth, no volatility growth seen in the graph below. Anything that causes a "reset" to a lower level, such as a large downturn in the portfolio, will weaken the eventual compound returns. This principle is strongly interconnected with the second factor, the value of avoiding large losses. These two factors go hand in hand. The power of compounding, crucial to successful long-term returns, can be better utilized when avoiding large losses.



Source: diyinvesting.org

2. The Value of Avoiding Large Losses

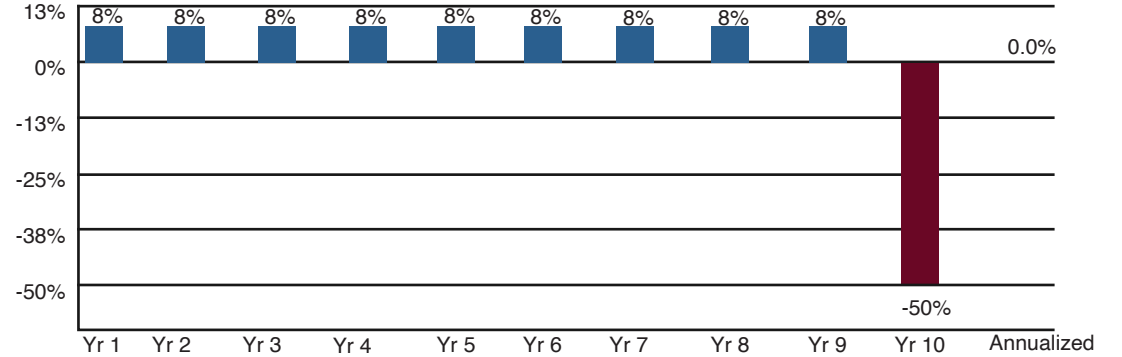
Large losses can be incredibly painful in the short-term, but their impact on the long-term success of investment returns is even more dramatic. Many studies show the value of avoiding large losses and how behavioral bias contributes to people frequently participating in large losses.

Research has shown that most individuals are risk avoiders when handling gains and are risk takers when dealing with losses (Tversky and Kahneman, "Judgment under Uncertainty: Heuristics and Biases," 1982). Tversky and Kahneman had people receive $1,000 with the choice of a guaranteed gain of $500 or a 50% chance of a $1,000 gain. Over 80% chose the $500 guarantee, with few willing to take the risk of additional gain. On the other hand, in the second part of the experiment, people were given $2,000. They were then given the alternatives of a 50% chance of losing $1,000 or a 100% chance of losing $500. Around 70% chose the chance of losing $1,000, with few unwilling to avoid the risk of a larger loss. The results of the experiment indicated people tend to do the following when it comes to investing: they don't let their profits run and they fail to cut their losses short. The reverse of this psychology is necessary to be a successful trader or investor. As Warren Buffett once famously said regarding the rules of investing: "Rule #1: Never lose money. Rule #2: Never forget rule #1."

It is of course challenging for investors to avoid large losses. It is hard-coded in our DNA. This behavioral bias is called the disposition effect. The disposition effect is a behavioral bias wherein an investor exhibits reluctance to realize losses, as seen in the aforementioend experiment. Investors tend to sell winners too early and ride losers too long, hoping that they might eventually turn into a gain. Studies by Shefrin and Statman (1985), Barberis and Xiong (2009), Odean (1998), and Weber and Camerer (1998), to name a few, have demonstrated this disposition effect evident in investors' behavior. This is despite the fact that large losses can occur much more quickly than large gains. As the Oracle of Omaha once said: "It takes 20 years to build a reputation and five minutes to ruin it. If you think about that, you'll do things differently."

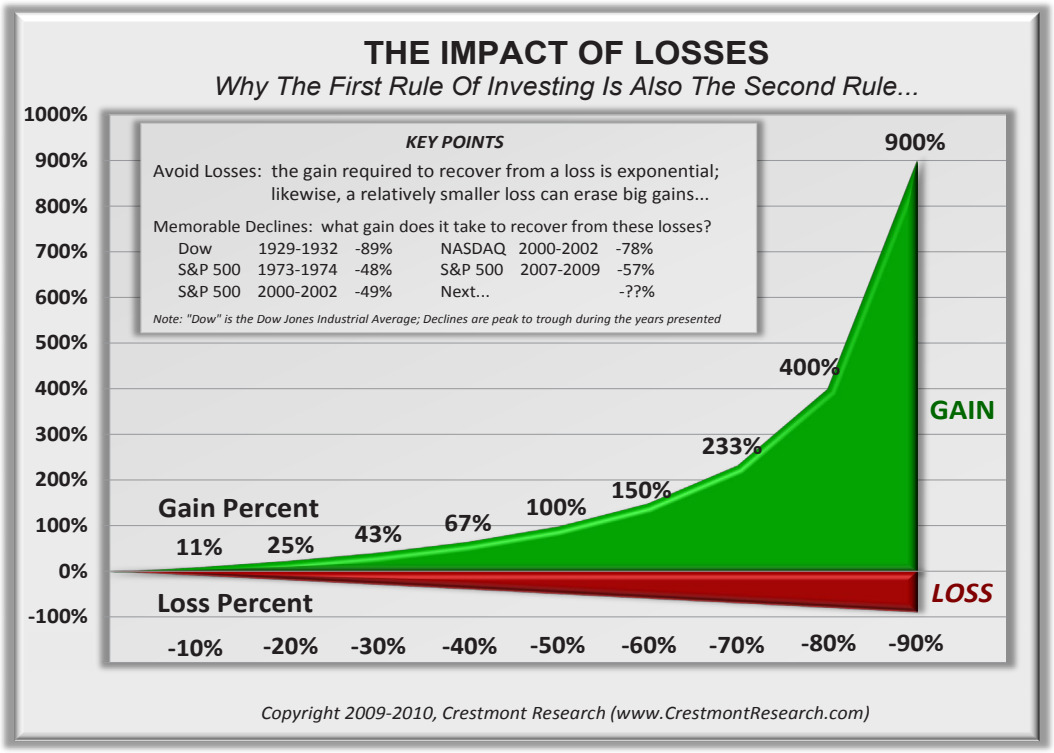
In the same vein as Buffett's quote, you could replace the word "reputation" with "portfolio," since large losses can quickly and disastrously wipe out years of investment growth. With that in mind, you SHOULD do things differently and always address and define risk so that large losses do not occur, or at least occur less frequently. For example, a solid 8% a year means you can double your money in 9 years (rule of 72). But if you take a 50% loss in year 10, you would be right back to where you started and the annualized return over those years would be 0%.

One Really Bad Year Can Erase Many Good Years of Gains



Source: PIMCO and Swan Global Investments

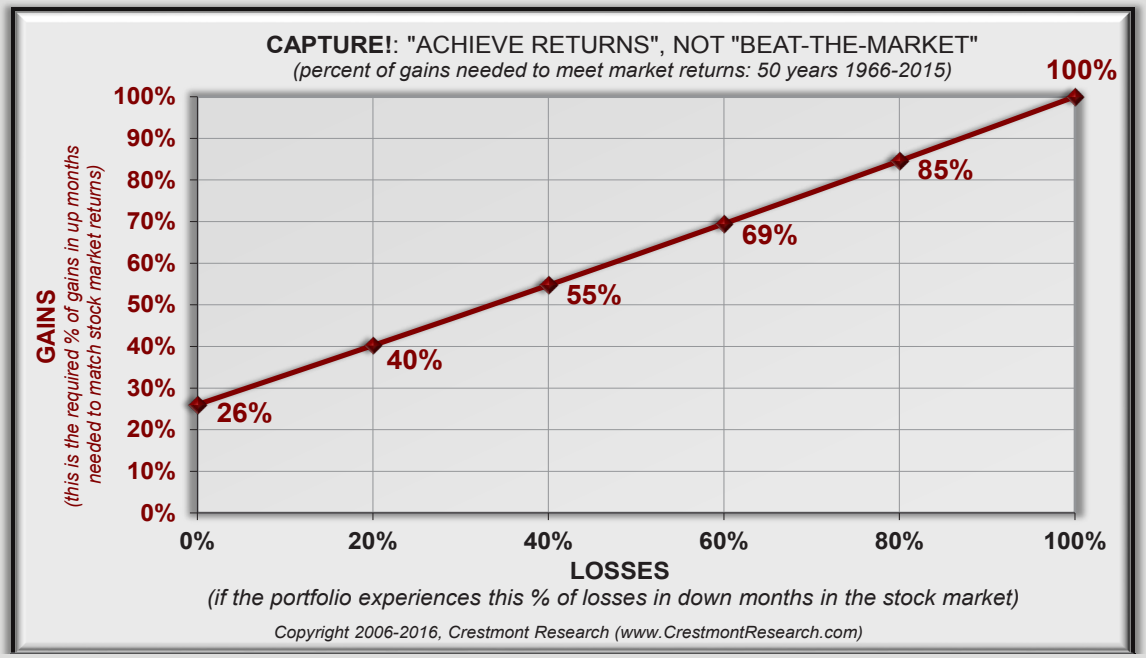
Crestmont Research, in a thought provoking white paper entitled "Half & Half: Why Rowing Works," graphically displayed the dynamic of the gains necessary to offset losses. As the losses increase, the required gain to recover your losses exponentially increases. The exponential power of large losses really starts to take effect after -30% to -40%, as seen below.



Source: Crestmont Research

One key takeaway from this graph: avoiding a loss is the equivalent of capturing a gain of greater magnitude. This is why investors should consider the statistical measurements of upside and downside capture for a strategy.

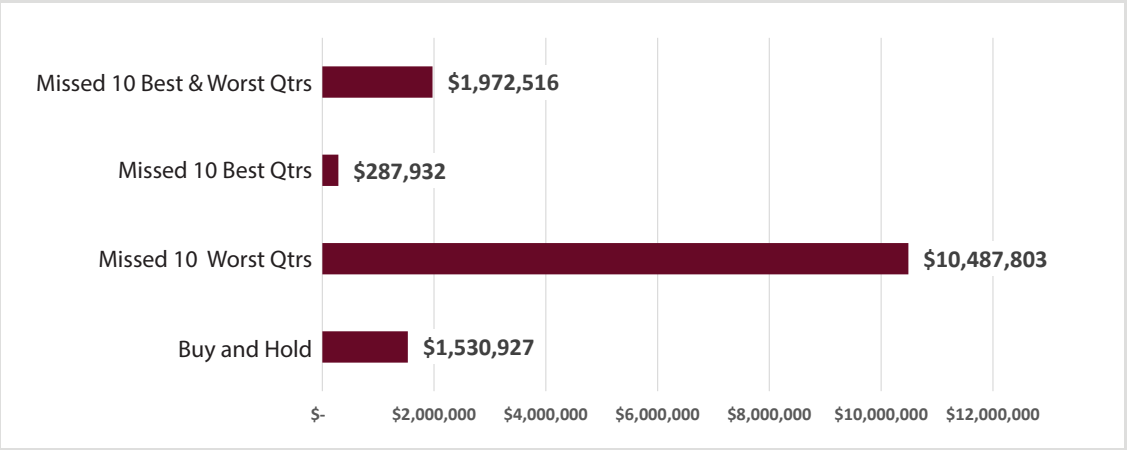
In many cases, too much focus is mistakenly put on the upside capture statistic. Although upside capture is necessary, it is not nearly as important as the downside capture of an investment strategy. Crestmont provides another graphic that illustrates this key point incredibly well and ties together these first two factors of compounding returns and avoiding large losses.



"Two volatility gremlins – the disproportionate impact of losses and the friction loss from the dispersion of returns – significantly reduce the compounding of returns. Many absolute return-oriented investment strategies recognize this dynamic and seek to enhance investors' compounded returns by providing a more risk-managed and consistent return profile. "Capture" is one way to measure and illustrate the effectiveness and benefit of this approach. Whereas the 'relative return' investor (tracking stock market indexes) will generally experience 100% of the downside and 100% of the upside to achieve market returns, the 'absolute return' investor only needs a fraction of the upside when downside losses are limited. This graph illustrates just how little of the upside is needed to match stock market returns over time and it demonstrates the way that many absolute return strategies exceed stock market returns without having to "beat-the-market" each year." - Crestmont Research

What this graph indicates is that if you were somehow able to miraculously avoid participation during down months in the stock market, you would only have needed to get 26% of the gains during the up months in order to match the market over that time (9.48% annualized return for the DJIA Total Return or 9.47% for the S&P 500 Total Return). If participation in the down months was 40%, then only capturing 55% in the up months would be needed to match the market. This is an astonishing realization to most investors and a definite reason to rethink the math behind investments. So many investors fall into the bad habit of hopping from one investment to another, chasing market performers trying to "beat the market". The key to successful long-term investing is not trying to find the hottest performer to get great up market participation, but minimizing or avoiding losses!

How big of a difference can avoiding large losses make over the long-term?



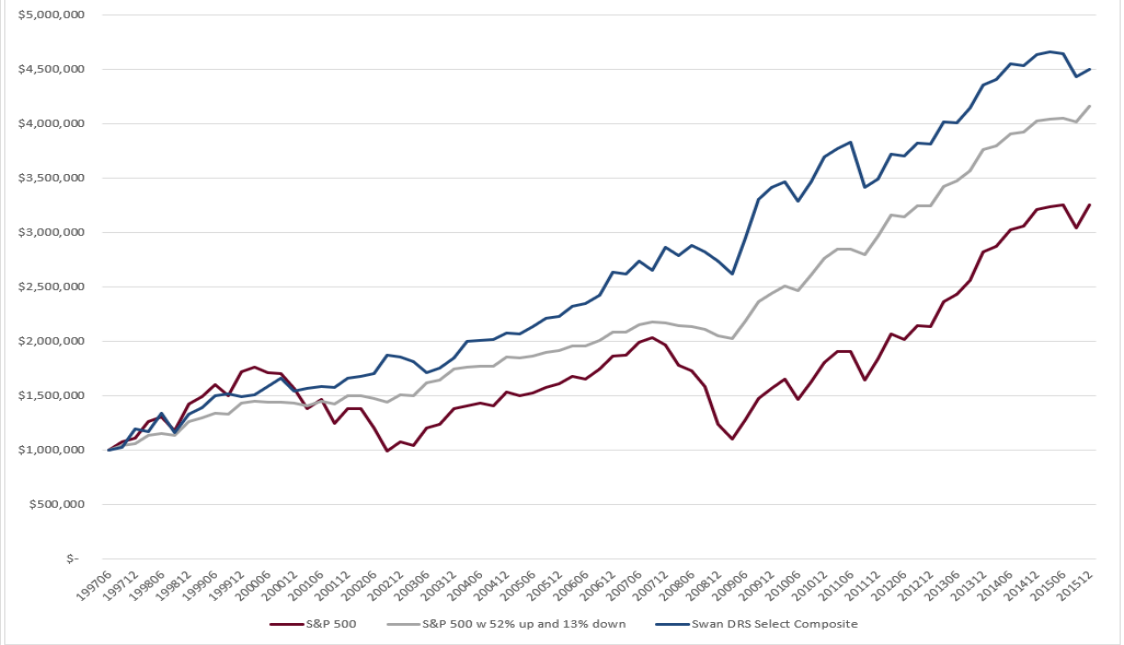
Source: Swan Global Investments. Data based on historic returns of the S&P 500 Total Return Index. Prior performance is not a guarantee of future results and there can be no assurance, and investors should not assume, that future performance will be comparable to past performance.

The "Passive vs Active Risk Management" chart above highlights the amazing impact avoiding large losses can have over time when compared to a buy and hold strategy. A $10,000 investment in the S&P 500 Index in 1970, if left alone, would have grown to an astounding $1.50mm by the end of 2019 (15,300%). However, if only the ten worst quarters in the Index were avoided across the 204 quarters over that time period, the final result would have been 89,570% more capital or $10.487mm (104,870% compared to 15,300%)! Avoiding periods of large declines can have an enormous impact on returns (and peace of mind). Although it is impossible for any equity strategy to completely miss the ten worst quarters in the market over a long timeframe, it is possible through proper hedging or other various strategies to miss out on participating in some of the downside associated with the worst performing quarters in the market, and thus, in essence, “miss” some of those quarters. Even if this means missing out on full participation in some of the best quarters, missing the worst quarters has a much greater impact on an investor’s bottom line. If all of the worst and best quarters were completely missed, an investor still ends up with an amount 22% higher than a buy and hold investor ($1.972mm). This is due to the power of compounding and avoiding the power of negative compounding.

Avoiding Large Losses and the Defined Risk Strategy

By design, Swan’s Defined Risk Strategy (DRS) was meant to minimize losses. This is especially important for those investors in the retirement stage who are drawing down their accounts to fund living expenses. And the DRS has been able to historically minimize losses by always being hedged. Across 264 months of the DRS since its inception through the end of 2019, the S&P 500 has had 34 months of losses greater than -3.5%, or around 13% of the time. The DRS by comparison has had only 17 or 6.6% of months with losses greater than -3.5% in a month, cutting the occurrences in half. As it relates to the DRS’s capture ratio of up/down quarters, the DRS has captured around 52% of positive quarters in the S&P 500 and 13% of negative quarters. This should and did lead to better results than the S&P 500 over the long-term and full market cycles.

S&P 500 TR vs Alternative Up/Down Capture



Source: Zephyr StyleAdvisor and Swan Global Investments. The S&P 500 Index is an unmanaged index, and cannot be invested into directly. Swan DRS returns are from the Select Composite, net of all fees. NOTE – this chart is for illustration purposes, not a guarantee of future performance. The charts and graphs contained herein should not serve as the sole determining factor for making investment decisions.

The Up/Down Capture chart above shows how the market has performed since the DRS's inception in 1997 versus the DRS and a hypothetical investment that systematically captures 52% of up quarters and 13% of down quarters. Notice this hypothetical shift in the mathematical returns nicely outperforms the S&P 500 most of the period after some underperformance during a bull market in the early years. The DRS itself has done even better than the hypothetical example. The hypothetical case was calculated by applying a 52% capture ratio to every up quarter and a 13% capture ratio to every down quarter, whereas in reality the DRS's capture ratios vary on a quarter to quarter basis—sometimes better, sometimes worse—and this of course varies its path.

Expanding the timeframe further out over its 22 years, the largest one-year calendar loss for the DRS equaled -7.74% (all results represented by the DRS Select Composite). The S&P 500's largest calendar year loss over that same timeframe was -37%. In fact, the S&P 500 had four years with annual returns worse than the DRS's worst loss year over the same time period.

Many times, the challenge with investors is getting them to think of investments in relation to their time horizon and goals. The longer the time horizon, the more important these math principles become. By the same token, emotions like fear and greed become less important. Crestmont's Ed Easterling states it this way:

> "Too often, investors are so focused on the task at hand that they can lose sight of taking the actions that are necessary to best achieve their goals. With investments, the goal is to achieve successful returns over time. We should not be distracted by a focus on this week or month; we need successful returns over our investment horizons—which often extend for a decade or two or more." (Crestmont Research)

How can an investor know whether an investment strategy will truly be able to avoid large losses? Managers can call themselves "risk-managed" or "tactical." but that does not mean their strategy will successfully avoid large losses. Managers can either actively structure a strategy to seek to avoid large losses by defining risk through a non-correlated asset such as options, or they can attempt to avoid large losses through market timing or passive diversification (undefined risk).

Based upon the examples discussed previously, it makes sense for the majority of an investor's portfolio to be constructed of investments that actively seek to avoid large losses. More aggressive strategies can be beneficial in the proper place, amount, and time, but the bulk of an investor's portfolio should not be in investments that can experience large losses. This is part of the shortcoming with today's popular portfolio management approaches based on Modern Portfolio Theory (the investment driving force behind fast-growing robo-advisors and target date funds). These approaches seek to put investors in a majority of passive index investments or active equity managers and another large percentage in fixed income investments, despite increasingly low or negative yields around the globe. Although some diversification is provided, both of these asset classes have experienced large losses in the past and likely will in the future. This approach doesn't take into consideration these various factors behind growing long-term wealth; traditional investment management needs to rethink the math and risk behind their investment strategies and think outside the (style) box.

3. Variance Drain (Volatility)

If someone were to tell you the market has averaged roughly 9.7% per year since 1928, you would want that investment, right? But how many years did the market actually have a return in the 9%-10% range? The answer might be surprising. Over the last 87 years, in only 2 years was the market return near its long-term average returning 9 or 10% and only 4 years within 2% (7.5-11.5). More often than not, the market's return in any given year was much greater or less than 9.7% and sometimes dramatically so. And that is the problem with averages. Averages, by their very nature, mask volatility. And when it comes to compounding returns, volatility and the sequence of returns is very important to the final outcome. In reality, not everyone wants an investment that grows at an average 9.7% if it involves multiple -40% to -80% drawdowns and sometimes decades of negative return in order to get it.

This is another reason why defining downside risk is so important. Lowering volatility is key to achieving better compound growth as volatility diminishes the rate at which an investment grows over the long-term. Volatility has a measurable negative effect on returns because of its impact on compounding.

This impact is known as variance drain or volatility drag. When two investments with the same average return are compared, the one with the greater volatility, or variance, all other things being equal, will have a lower compound return. This is due to the effect of negative compounding on the more volatile investment.

The concept of variance drain or volatility drag comes from the term "arithmetic mean - geometric mean inequality" and was detailed in a 1995 paper titled "Variance Drain - Is Your Return Leaking Down the Variance Drain?" by Tom Messmore. Messmore observed that the more variable a given asset's return is, the greater the difference between the arithmetic and geometric returns. Arithmetic mean is the average of a set of numerical values, calculated by adding together and dividing by the number of terms in the set. Geometric mean is defined as the value of a set of numbers by using the product of their values, as opposed to the arithmetic mean which uses their sum.

A formula of variance drain looks like this:

$$r_g \approx r_a - \frac{1}{2}\sigma^2$$

rg = geometric return, ra = arithmetic return, = its variance

This formula shows that the variance of returns "drains" the arithmetic average returns to produce the smaller, realized, compound returns (Decker, Robert. "The Variance Drain and Jensen's Inequality").

As an example, let's say an investment is purchased for $100. At the end of the first year, its value has doubled to $200, a 100% gain. In the second year however, the value undergoes a 50% loss and drops to $100.

The arithmetic return over 2 years is thus 25% or = (100% - 50%) / 2

The geometric return over 2 years is 0% or = $100 (final value) - $100 (starting value)

In this example, the variance drain is 25% = 25% (arithmetic average) - 0% (geometric average).

Why should this matter for investors? In order to optimally take advantage of the power of compounding, investors must avoid large losses and the exponential growth needed to recover from a loss. And in addition, investors must avoid the negative power of compounding by seeking to lower volatility drag as best as possible. Exhibit 9 shows this impact across various scenarios.

Again, we can see how these factors all tie together in their mathematical application within an investor's portfolio: compounding, avoiding large losses, and now volatility.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6
Arithmetic Annual Return	10%	10%	10%	10%	10%	10%
Standard Deviation (Volatility)	0%	10%	20%	30%	40%	50%
Geometric Annual Return	10%	9.60%	8.30%	6.03%	2.58%	-2.42%
Starting Funds	$100,000	$100,000	$100,000	$100,000	$100,000	$100,000
Ending Funds	$259,375	$250,156	$221,935	$179,629	$129,073	$78,278
Total 10 Year Return	159%	150%	122%	80%	29%	-22%

Source: Tyton Capital Advisors, "Low Charges And High Volatility: How To Erase Your Returns"

Compounding, whether negative or positive, is the common thread throughout all of them.

In addition, volatility can lead to detrimental investor behavior as seen from Dalbar's annual Quantitative Analysis of Investor Behavior report.

	Average Equity Fund Investor	Average Fixed Income Fund Investor	Average Asset Allocation Fund Investor	S&P 500	Bloomberg Barclays Aggregate Bond Index	Inflation
20 Year	3.88%	0.22%	1.87%	5.62%	4.55%	2.17%
10 Year	9.66%	0.70%	4.53%	13.12%	3.48%	1.82%
5 Year	3.96%	-0.40%	1.50%	8.49%	2.52%	1.56%
3 Year	5.58%	-0.11%	1.84%	9.26%	2.06%	2.04%
12 Month	-9.42%	-2.84%	-6.97%	-4.38%	0.01%	1.93%

Source: "Quantitative Analysis of Investor Behavior - Advisor Edition, 2018," DALBAR, Inc. www.dalbar.com. Returns are for the period ending December 31, 2018.

The first columns under of “Investor Returns” detail the actual experiences of average investors. Too often fear and greed lead to poor decision-making and thus underperformance. As a result, we see that investors in equities have vastly underperformed the S&P 500. Not surprisingly, the periods of the most acute underperformance for investors occur during some of the most volatile times in the market. This is largely due to the tendency of investors to get in and out of the market at the wrong time. The market goes down to a point that many investors can’t take the pain anymore and they sell their positions. And then the market goes back up but investors don’t believe the rally for a while and then buy at a higher level. This habitual unintended practice of “sell low, buy high” drives this drastic underperformance.

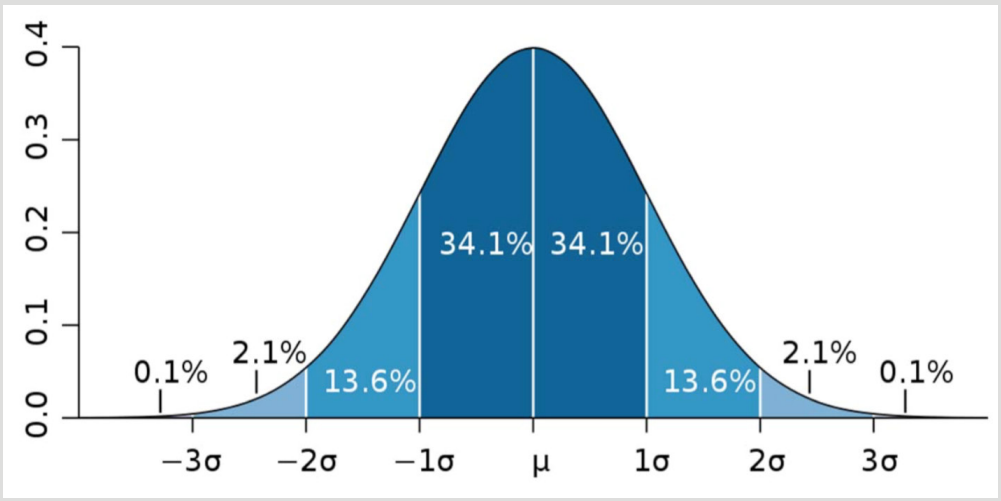
The final outcome is greatly impacted by the volatility of the experience along the way. Whether or not an investor can stay the course with an investment or survive withdrawing money from their account depends on the volatility of the portfolio. High volatility portfolios tend to lead to poor, emotionally driven decisions because investors have a difficult time staying invested.

Volatility, usually accompanied by fear, tends to lead to rash decisions. It is unlikely that the average investor will ever completely purge fear from their decision-making. Therefore, the next-best solution is to invest in a strategy that lowers volatility in the first place and helps to minimize some of the fear by establishing a defined amount of risk. If a strategy can lower volatility without sacrificing too much return over the long-term, it can likely lead to a better experience and outcome for the investor. If you can’t stick with a portfolio, it doesn’t matter what its compound annual growth rate is; you will never “experience” that return! This is why investors should consider the fourth and next factor when choosing an investment; its distribution of return.

4. Distribution of Returns

Utility theory posits this: (1) investors prefer more return to less and (2) investors dislike uncertainty (Bollen, "Measuring the Benefits of Options Strategies in Portfolio Management"). Investors should naturally then prefer portfolios with asymmetric return distributions; in particular, distributions that have very low probability of extremely bad outcomes and consistency of returns. But what is a return distribution and why should an investor care what an investment’s return distribution looks like?

A return distribution is a probability distribution or a statistical function that shows all the possible values and likelihoods that a random variable can take within a given range. The area under a normal distribution, or the familiar looking bell-shaped curve, denotes the probability of the returns. The probability decreases to the left or right of the mean. There are four moments to a return distribution: return, standard deviation or volatility, skewness, and kurtosis. When analyzing historical returns of an investment, a return distribution helps assess the likelihood of where returns might fall and to assess the asset’s level of risk and return potential.



Source: Wikipedia

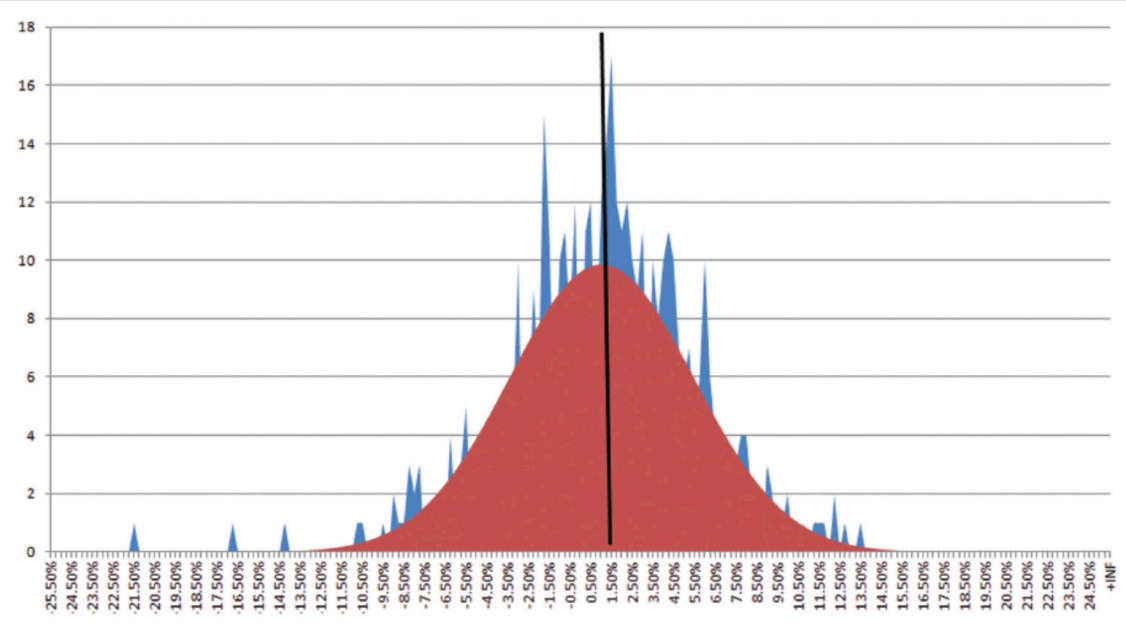
The distribution above is an idealized, "normal" distribution where outcomes fall into a very predictable pattern. The above shows, for example, that 68.2% of occurrences should fall within one standard deviation. A normal distribution is also symmetrical, meaning both the count and scale of observations above and below the mean occur with an equal probability. However, stock market returns do not often fit into this idealized pattern. Empirical observations for equities have shown that the distribution of returns is often characterized by left-skewed distributions and either pointy or flat distributions. A left-skewed distribution denotes that outlier events tend to occur on the downside. The tallness or flatness of the distribution is derived from the kurtosis of a distribution; this refers to how tightly packed around the mean the deviations will be. This table highlights each moment within a distribution of returns and their characteristics:

Table 1: Return Distribution Characteristics

NAME	MOMENT	COMMON NAME	CHARACTERISTIC	PREFERENCE
Mean	First	Expected Return	Balance point of the area under the distribution	Higher values with higher moments constant
Standard Deviation (variance)	Second	Volatility	Measure of the width (dispersion)	Lowest value to meet requirement
Skewness	Third	Fat tail	Measure of symmetry	Positive
Kurtosis	Fourth	Fat tail	Measure of shape, tall or flat	Negative downside, positive upside (Note: Kurtosis for a normal distribution is 3)

Source: Evestment

Distribution of Monthly Returns for the S&P 500



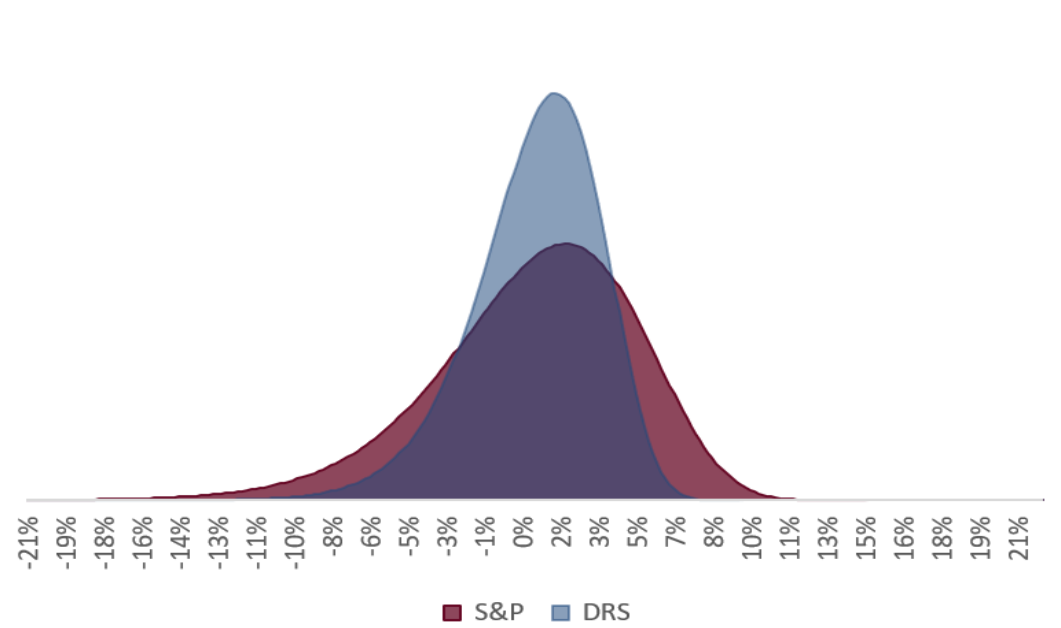
Source: Evestment

Instead, equity markets generally follow a fairly normal distribution pattern but with negative skewness, high kurtosis, and large fluctuations within the fat tails (tail risk events). What does that mean, in plain English? It means that most of the time individual months will fall fairly close to their long-term average, but when things go bad in the markets, they go really bad. The worst of the bad months are more extreme than the best of the good months. Volatility, when it does happen, tends to be driven by the extreme tail events.

So if that is what the market gives us as a distribution of return, what can we do as investors? We should try to tilt the odds in our favor by pursuing a distribution of returns that has a healthy number of upside observations and structurally limits the downside, especially the far left tail occurrences, as much as possible. Ideally then, investors should consider all four moments in an investment's return distribution and find investments that have these optimal distribution patterns.

One possibility is hedged equity or options-based equity strategies. This is exactly what the DRS has been able to do; shift the return distribution into a more optimal pattern compared to traditional equities. Let's take a look at the difference between the DRS and the S&P 500:

Smoothed Monthly Return Distribution since July 1997 Inception of DRS Select Composite

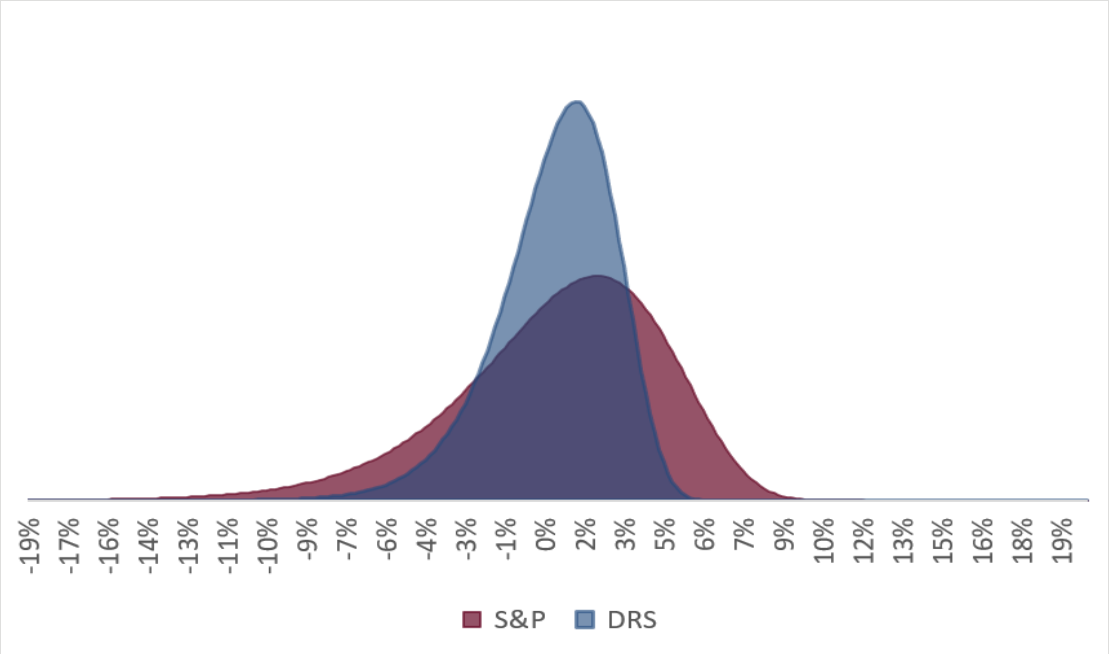


Source: Swan Global Investments; S&P 500 Total Return monthly data from Morningstar. DRS returns are from the Select Composite, net of all fees. Note – This chart is for illustration purposes not a guarantee of future performance.

First, let's take a look at the return distribution of the S&P 500 Total Return, represented by the red curve, since July 1997, when the DRS began. Notice how the red curve looks fairly similar to a normal distribution, but is skewed to the left a little bit and has a fatter left tail (more big down moves). The red curve is showing that it has a lot more occurrences where the returns were worse than -5% or greater than +5% compared to the blue curve of the DRS over this time period. Contrast this with the return distribution for the DRS. With this visual you can see how the DRS has historically cut off the far left and right tails and squeezed more occurrences into the middle of the distribution, giving it the higher kurtosis or pointedness in the curve but with a much smaller left tail (less large losses).

If we show a histogram from 2004 when the DRS formalized its philosophy to not make market-timing calls, the following graph shows all the occurrences that drive these curves:

Histogram: Distribution of Returns since 2004 of DRS Select Composite



Source: Swan Global Investments; S&P 500 Total Return monthly data from Morningstar. DRS returns are from the Select Composite, net of all fees. Note – This chart is for illustration purposes not a guarantee of future performance.

Again, notice the lack of DRS occurrences in the far left and far right columns, while the lighter shaded blue shows the additional occurrences that take place in the center in the -1% and 1% buckets. Why should an investor consider the return distribution of a strategy? An investor should want greater frequency of return in the median and a removal of occurrences in tail events. It is in the left tail where fear takes over, and it is in the right tail where greed infects an investor's thinking. If return distribution is consistent across periods of time and the reasons for return are established as driven by process and not decisional-driven timing luck/skill, an investor can get a good idea whether a strategy might provide an optimal means of return over time. It should paint a clear picture of whether a strategy actually avoids large losses to lessen the negative compounding effect and take advantage of lower volatility and steadier, compounding growth.

COMPARING THE FOUR BASIC WAYS TO INVEST IN EQUITIES

Reliance by long-term investors on equity as one of the main engines of growth in a portfolio has been empirically verified through numerous studies. Equities tend to grow over time, albeit with considerable risk. The table below shows how real returns in equities over long periods of time have generally led to tremendous growth in most developed nations around the world. However, the table also shows that across various timeframes, if you happen to have bad luck and invest during a worst-case scenario, returns can be devastatingly bad. No country listed had positive real returns between the 1, 3, and 10-year worst-case returns and only three countries had positive returns for the 20-year worst-case scenario.

The main takeaway from this information should be that equities over the long-term tend to be a great source of growth, but at tremendous risk of large losses along the way. If equities can be harnessed for its growth potential while avoiding the large losses and worst-case scenarios, it could become an even better source of return and one that deserves to be a core holding for those that don't have 40 or 50 years to stay invested.

		Worst Case Scenario						
	Annualized Return (1900-2012)	1-Year	3-Year	10-Year	20-Year	30-Year	40-Year	50-Year
Australia Real USD	7.42	-52.71	-50.09	-31.01	48.61	198.55	324.98	827.40
Equal Weighted USD	**6.73**	**-48.14**	**-56.99**	**-53.44**	**-13.22**	**119.11**	**197.32**	**592.04**
South Africa Real USD	6.51	-43.30	-53.57	-51.78	-32.71	117.35	162.32	394.99
US Real	**6.26**	**-37.59**	**-59.52**	**-33.27**	**19.14**	**132.49**	**350.62**	**855.57**
Canada Real USD	5.77	-46.48	-59.29	-33.86	15.00	105.67	317.40	733.95
New Zealand Real in US$	5.75	-49.73	-45.71	-28.77	-12.56	65.62	79.93	298.86
Sweden Real USD	5.64	-49.60	-77.23	-72.46	-72.79	-39.56	4.60	71.85
Denmark Real USD	5.58	-50.48	-64.35	-50.32	-11.10	-17.94	86.75	233.64
Finland Real in US$	5.35	-75.29	-87.17	-85.55	-62.25	-37.32	-16.00	84.96
UK Real USD	5.23	-53.94	-66.55	-45.32	-30.89	39.81	73.16	111.40
Netherlands Real USD	5.20	-69.00	-79.62	-71.02	-52.54	-52.31	-25.95	78.57
Switzerland Real USD	5.17	-42.94	-73.03	-72.63	-57.14	54.36	103.47	218.62
World Real USD	**5.01**	**-40.98**	**-53.03**	**-58.12**	**-30.01**	**47.65**	**127.31**	**284.95**
Norway Real USD	4.53	-63.27	-81.93	-70.01	-44.74	-40.40	-14.37	-3.69
Japan Real USD	4.20	-91.84	-98.65	-98.52	-97.09	-97.21	-93.86	-71.07
Ireland Real USD	4.14	-66.78	-65.14	-63.84	-59.23	25.05	34.23	45.67
Spain Real USD	3.53	-50.69	-71.15	-85.94	-79.67	-57.36	-61.99	-55.90
Germany Real USD	3.35	-79.68	-93.41	-96.54	-97.99	-99.16	-99.13	-88.50
Belgium Real USD	3.02	-50.19	-71.60	-76.12	-64.93	-53.60	-62.66	-35.16
France Real USD	2.91	-77.54	-83.65	-75.21	-74.39	-66.93	-74.88	-61.52
Italy Real USD	1.94	-62.43	-79.93	-85.63	-77.57	-54.65	-60.28	-29.10
Austria Real USD	-0.17	-73.26	-92.34	-96.83	-96.63	-98.57	-99.42	-99.10
Average	4.68	-57.99	-71.09	-65.28	-44.76	8.67	56.98	199.47
Percentage Negative	5%	100%	100%	100%	86%	55%	45%	36%

based on annual data from Dimson, Marsh & Staunton

Source: Patrick O'Shaughnessy, The Investor's Field Guide: Dangers of Portfolio Patriotism; worst case scenarios are cumulative returns

There are four basic approaches to investing in liquid equities:

1. Passive investing: You can buy the market through an index ETF or fund and get its unknown returns and its unknown pattern of returns. Historically, someone can get an idea of what a passive approach is probably going to look like as seen in the prior image of the distribution of the U. S. equity market, which looks fairly similar across longer time periods. It can vary greatly, but an investor is likely to get double-digit gains or losses in most years, an occasional but fairly rare flat year, and over time, mid to high single digit returns. It is important to note though that on a sufficiently long enough timeline, the probability of being a completely passive investor goes to zero. If you are planning to invest for an objective other than buying and holding forever, you have to make decisions eventually about when and how much to invest and when and how much to withdraw, as well as your start period and end period; all active decisions (Credit: Druce Vertes, StreetEye.com).

2. Active investing: You can buy an investment that is actively managed and get its unknown returns and its unknown pattern of returns. It's very possible to pick a great manager based on its track record and then the investment doesn't perform well going forward. Studies have shown that over 80% of active managers underperform their benchmarks on a 5-year and 10-year basis, both domestic and international equity, as seen below. It is extremely difficult, if not impossible, to consistently pick good active managers. Logically, it is easy to deduce that if a manager does beat its benchmark in one period it is highly unlikely to beat it the next period based upon these statistics.

Percentage of U.S. Equity Funds Outperformed by Benchmarks (April 2010 - March 2020)

Fund Category	Comparison Index	1-Year (%)	3-Year (%)	5-Year (%)	10-Year (%)
Morningstar Large Blend	S&P 500	79%	85%	93%	94%
Morningstar Mid-Cap Blend	S&P MidCap 400	42%	48%	75%	81%
Morningstar Small-Cap Blend	S&P SmallCap 600	49%	66%	87%	94%

Percentage of International Equity Funds Outperformed by Benchmarks (April 2010 - March 2020)

Fund Category	Comparison Index	1-Year (%)	3-Year (%)	5-Year (%)	10-Year (%)
Morningstar Foreign Large Blend	MSCI EAFE Index	66%	81%	81%	86%
Morningstar Diversified Emerging Markets	MSCI EM	54%	69%	65%	56%

Percentage of Fixed Income Funds Outperformed by Benchmarks (April 2010 - March 2020)

Fund Category	Comparison Index	1-Year (%)	3-Year (%)	5-Year (%)	10-Year (%)
Morningstar Intermediate Core	Bloomberg Barclays U.S. Aggregate	90%	93%	91%	72%
Morningstar High Yield Bond	Barclays US Corporate High Yield	62%	83%	92%	95%

Source: Zephyr StyleADVISOR, Swan Global Investments

3. Tactical equity investing: Tactical investing is another form of active investing, whether tactical active or tactical passive. You will still get unknown returns and an unknown distribution of returns as well. Like some active managers, a tactical manager can have years of great timing and outperformance and yet this means nothing for its future return.

With tactical managers, there is this risk of the unknown or unpredictability of outcome. As some active managers are just closet indexers or perform similar to their benchmark indices, this risk tends to be greater for tactical managers. Timing, stock selection, or tactical decisions could have led to excellent performance for many years and then the investment doesn't perform well going forward. In addition, tactical managers don't re-structure the return distribution of equities, but create an entirely new distribution of returns; one in which the return can be completely the opposite of the concurrent market return based on the manager's tactical decisions and timing. Uncertainty, active selection, and timing risk can play a big role in whether an investor achieves successful long-term returns with this approach.

So, if it is extremely hard to beat or achieve market return through picking an active or tactical manager and passively tracking the market means participating in all its volatility and potential for large drawdowns, how can investors think outside the box and participate in equity growth with more certainty and less volatility? There is a fourth approach: hedged equity.

4. Hedged equity: Hedged equity involves investing in equities while also hedging that exposure with non-correlated assets such as put options. Investing in a proven hedged equity strategy such as the DRS gets exposure to the long-term growth of equities, while always remaining hedged to protect against large downside losses. By using options to re-structure the return distribution, an investor can minimize the uncertainty and unknown that normally comes with equity returns. It is important to note, however, that not all hedged equity strategies are created equal. Some choose to hedge with different instruments that might not truly be non-correlated, while some choose when and if to hedge the underlying portfolio. These approaches are more of a mixture of tactical investing and hedged equity.

DRS Select Composite: July 1997 - December 2019

	Swan DRS	S&P 500	60% S&P 500 40% Barclays U.S.
Annualized Return since Inception	8.08%	7.94%	7.11%
Standard Deviation since Inception	9.35%	11.93%	8.92%
Beta since Inception	0.32	1.00	0.59
Sharpe Ratio	0.65	0.40	0.57

Source: Swan Global Investments. Note – This chart is for illustration purposes not a guarantee of future performance. The charts and graphs contained herein should not serve as the sole determining factor for making investment decisions.

So what have the results been for the DRS? How has the DRS done at avoiding some of the large losses in the S&P 500 over its 21-year track record? Does the math behind an approach that tends to trail in bull markets actually support this approach versus traditional equity returns? Here is a summary comparison:

The benefits of compounding growth and avoiding large losses become more and more apparent over time and over full market cycles. Even in bull markets like the last one that lasted several years that the strategy can underperform the S&P 500 and yet still mathematically provide better returns over a longer time period and full market cycle.

How does the DRS do this? The process is a fairly simple four-step process:

1. Establish equities; passive and invested at all times
2. Create hedge to define risk; buy long-term put options (LEAPS)
3. Seek to generate return through market-neutral option strategies
4. Monitor and adjust; rebalance and re-hedge the portfolio regularly to take advantage of large sell-offs (this re-hedge has the ability to add additional equity shares to further compound future market growth)

This process establishes a predictable expected return band on a one-year timeframe when compared to passive equities. On the next page, there are the worst and best case scenarios on a rolling return monthly close basis since DRS inception in July 1997 for the DRS, S&P 500 Total Return, and a 60/40 portfolio (60% S&P 500, 40% Barclays U.S. Agg Bond Index):

Rolling Period	S&P 500: July 97-Dec 2019				Swan DRS: July 97-Dec 2019				DRS vs S&P 500
	Current Rolling Return	Average Rolling Return	Worst Rolling Return	Best Rolling Return	Current Rolling Return	Average Rolling Return	Worst Rolling Return	Best Rolling Return	% of Occurrences DRS Outperformed
Rolling 1 Year	31.49%	8.58%	-43.32%	53.62%	13.94%	7.84%	-7.74%	38.25%	34.36%
Rolling 2 Year	12.13%	7.60%	-26.08%	37.22%	2.53%	7.62%	-0.80%	23.95%	38.06%
Rolling 3 Year	15.27%	6.84%	-16.09%	25.56%	5.22%	7.47%	1.31%	19.18%	45.11%
Rolling 4 Year	14.44%	6.39%	-9.76%	22.42%	6.30%	7.40%	2.13%	12.90%	54.26%
Rolling 5 Year	11.70%	6.40%	-6.63%	23.00%	4.39%	7.48%	2.99%	12.89%	57.35%
Rolling 6 Year	12.03%	6.66%	-1.13%	21.72%	4.74%	7.57%	4.50%	11.14%	63.82%
Rolling 7 Year	14.73%	6.65%	-3.85%	17.27%	6.06%	7.66%	4.51%	10.81%	67.38%
Rolling 10 Year	13.56%	6.03%	-3.43%	16.67%	5.34%	7.79%	5.31%	10.67%	66.89%
Rolling 15 Year	9.00%	6.49%	3.76%	10.53%	7.04%	7.67%	6.76%	9.38%	64.84%
Since Inception	7.94%	7.53%	-0.19%	30.16%	8.08%	10.75%	7.61%	33.76%	89.19%

Rolling Period	60/40 Portfolio: July 97-Dec 2019				Swan DRS: July 97-Dec 2019				DRS vs 60/40
	Current Rolling Return	Average Rolling Return	Worst Rolling Return	Best Rolling Return	Current Rolling Return	Average Rolling Return	Worst Rolling Return	Best Rolling Return	% of Occurrences DRS Outperformed
Rolling 1 Year	22.18%	7.06%	-27.65%	34.50%	13.94%	7.84%	-7.74%	38.25%	34.36%
Rolling 2 Year	9.23%	6.54%	-14.59%	24.76%	2.53%	7.62%	-0.80%	23.95%	38.06%
Rolling 3 Year	10.87%	6.22%	-7.24%	18.45%	5.22%	7.47%	1.31%	19.18%	45.11%
Rolling 4 Year	10.22%	6.01%	-4.05%	16.11%	6.30%	7.40%	2.13%	12.90%	54.26%
Rolling 5 Year	8.37%	6.04%	-2.26%	15.85%	4.39%	7.48%	2.99%	12.89%	57.35%
Rolling 6 Year	8.74%	6.21%	1.69%	15.08%	4.74%	7.57%	4.50%	11.14%	63.82%
Rolling 7 Year	9.96%	6.21%	-0.12%	12.31%	6.06%	7.66%	4.51%	10.81%	67.38%
Rolling 10 Year	9.77%	5.92%	0.42%	11.56%	5.34%	7.79%	5.31%	10.67%	66.89%
Rolling 15 Year	7.30%	6.16%	4.69%	8.14%	7.04%	7.67%	6.76%	9.38%	64.84%
Since Inception	7.11%	7.43%	2.79%	22.21%	8.08%	10.75%	7.61%	33.76%	89.19%

Source: Swan Global Investments and Zephyr StyleAdvisor. Data based on historic returns of the S&P 500 Total Return Index and Swan DRS Select Composite net of fees, from 7/1997 to 12/31/2019, and assume no portfolio withdrawals. Results in the table are the best, worst, and average annualized returns, based on month-end returns, for every rolling period listed within the overall time frame of July 1st, 1997 to December 31st, 2019. Prior performance is not a guarantee of future results and there can be no assurance, and investors should not assume, that future performance will be comparable to past performance.

This information shows how the DRS, by seeking to avoid large losses and lower volatility, is able to have a more predictable, stable rolling return with much better worst case scenarios than the S&P 500 or a 60/40 portfolio. In fact, since inception the DRS has had a positive two-year rolling return 98% of all rolling months, compared to 72% of the time for the S&P 500 over the same time period. All other rolling returns above have been positive returns on a 3, 4, 5, 6, 7, and 10-year basis for the DRS. Not the case for the S&P 500, with substantially negative returns on all of the same timeframes. This consistency and the benefits of it for those seeking to grow their assets or those seeking to withdraw income from their portfolio is further described on our website in Marc Odo's white paper "The Retirement Conundrum: Untying the Gordian Knot". Of course, past returns are no guarantee of future results but they do give a picture into how these mathematical principles apply over full market cycles.

The table below gives the specific details of how the math adds up over time, starting with the DRS's first full year through the end of 2019. Even though the arithmetic average return has been the same for the DRS and the S&P 500, the lower volatility drag and ability to limit participation in down markets has led to a higher geometric return for the DRS. The table shows the four mathematical principles in action over time.

Year	Return %	Swan Defined Risk Strategy (net)	Return %	60 S&P / 40 Agg	Return %	S&P 500
Initial Value		1,000,000		1,000,000		1,000,000
1998	11.6%	1,115,525	21.0%	1,209,796	28.6%	1,285,766
1999	12.3%	1,252,250	12.0%	1,354,939	21.0%	1,556,350
2000	3.2%	1,291,978	-1.0%	1,341,469	-9.1%	1,414,639
2001	7.5%	1,388,421	-3.7%	1,291,679	-11.9%	1,246,512
2002	12.2%	1,558,059	-9.8%	1,164,833	-22.1%	971,027
2003	-0.6%	1,547,944	18.5%	1,380,088	28.7%	1,249,561
2004	12.3%	1,738,075	8.3%	1,494,623	10.9%	1,385,540
2005	7.5%	1,867,883	4.0%	1,554,474	4.9%	1,453,596
2006	18.1%	2,206,669	11.1%	1,727,262	15.8%	1,683,183
2007	8.8%	2,401,102	6.2%	1,834,762	5.5%	1,775,654
2008	-4.5%	2,293,097	-22.1%	1,429,941	-37.0%	1,118,697
2009	25.0%	2,866,405	18.4%	1,692,986	26.5%	1,414,749
2010	8.1%	3,098,467	12.1%	1,898,269	15.1%	1,627,859
2011	-5.4%	2,931,650	4.7%	1,987,284	2.1%	1,662,231
2012	9.0%	3,195,671	11.3%	2,212,040	16.0%	1,928,246
2013	14.3%	3,653,773	17.6%	2,600,398	32.4%	2,552,767
2014	6.5%	3,892,107	10.6%	2,876,487	13.7%	2,902,205
2015	-2.9%	3,778,226	1.3%	2,913,445	1.4%	2,942,369
2016	9.6%	4,140,547	8.3%	3,155,429	12.0%	3,294,276
2017	10.8%	4,588,913	14.2%	3,603,821	21.8%	4,013,468
2018	-7.7%	4,233,659	0.0%	2,645,581	-4.4%	3,837,504
2019	13.9%	4,823,626	22.2%	4,299,786	31.5%	5,045,800

Geo Compound Return:	7.4%		6.9%		7.6%
Arith Average Return:	7.6%		7.0%		8.5%
Standard Deviation:	9.2%		8.9%		14.8%
Cumulative Return:	382.4%		330.0%		404.6%

Source: Zephyr StyleAdvisor, Swan Global Investments; begins first full year, 1998. The Barclays U.S. Aggregate Bond Index and the S&P 500 Index are unmanaged indices, and cannot be invested into directly. Past performance is no guarantee of future results. DRS results are from the Swan Defined Risk Select Composite, net of fees, as of 12/31/2019.

CONCLUSION

It is time for investors to rethink how they view investment returns and what truly matters when investing in equities.

1. Compound growth matters
2. Avoiding large losses matters
3. Volatility matters
4. Distribution of return matters

In summary, there are four basic approaches to investing in equities. Passive investing is exposed to left-tail risk and high volatility. Active and tactical investing attempts to try to change the distribution of returns by making market calls, either from the bottom-up (active) or top-down (tactical). The DRS is structured to change the distribution of returns. Mathematics is rational; investing in equities can be irrational at times. By rethinking the math behind investment returns, investors should come to the rational conclusion that there is a viable alternative to buy-and-hold equity investing or irrationally jumping from one active manager to another in search of the hottest stock picker or market-timer. Anyone invested in equities for the long-term should consider hedged equity and the DRS. The math behind changing a return pattern, as hedged equity and the DRS seeks to do, supports this alternative approach to equity investing as a better way to achieve long-term returns. It is of vital importance that today's investors study and understand the mathematical principles behind investment returns in order to avoid behaviorial biases and to help them find the best possible solutions for reaching their financial goals.

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All data used herein; including the statistical information, verification and performance reports are available upon request. The S&P 500 Index is a market cap weighted index of 500 widely held stocks often used as a proxy for the overall U.S. equity market. Indexes are unmanaged and have no fees or expenses. An investment cannot be made directly in an index.

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Our simple, yet innovative investment philosophy is the foundation of our Defined Risk Strategy, a rules-based, multi-asset hedged equity strategy, with a track record of seeking to generate consistent returns while defining, or limiting, downside risk to improve investment outcomes and managing risk for irreplaceable capital through full market cycles (bull and bear).

Swan Global Investments is an asset manager headquartered in Durango, Colorado, with offices in Puerto Rico and Tampa, Florida.

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